Stability of opinion formation PDE model based on expanded non-local perceptual kernel

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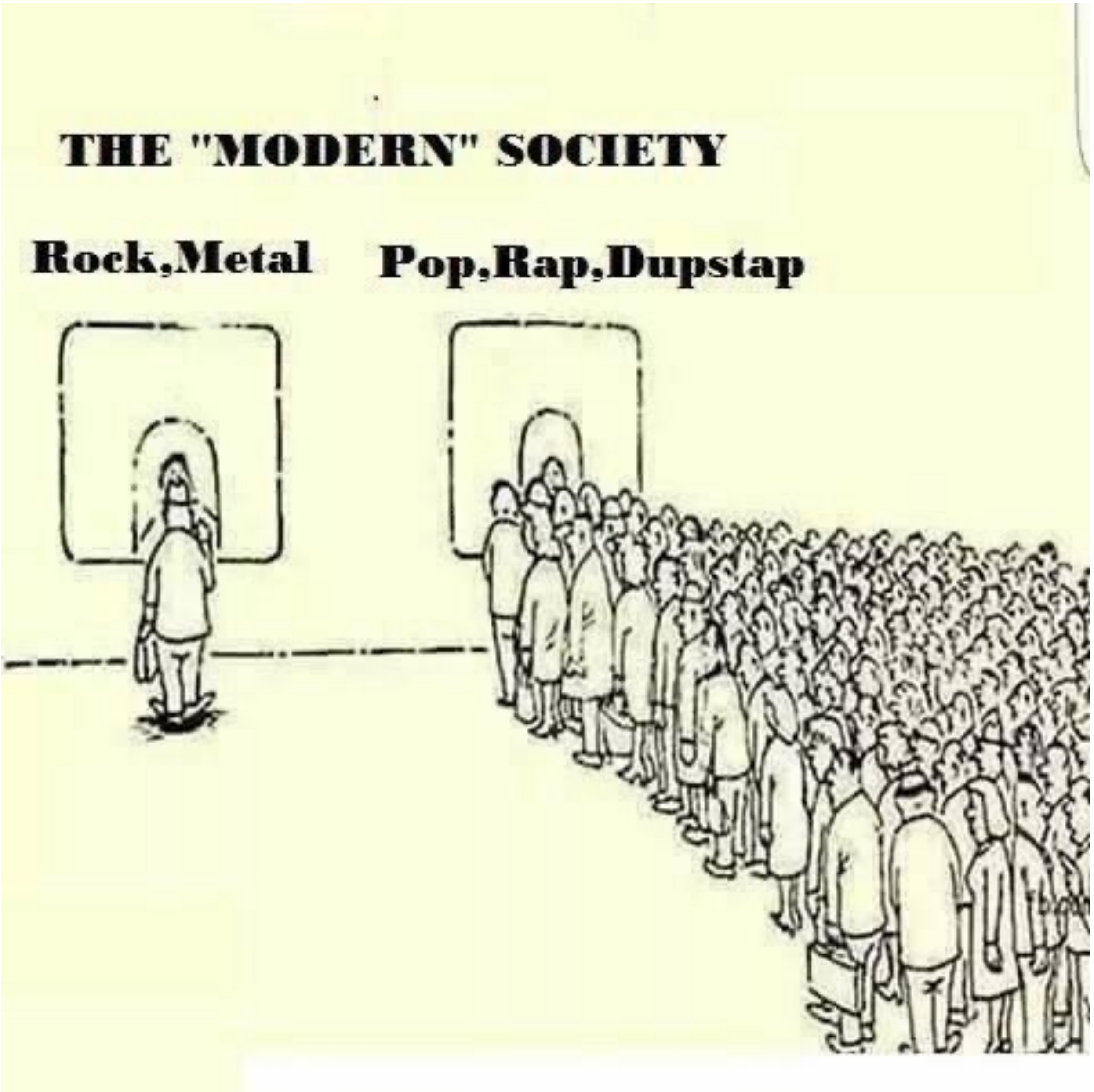
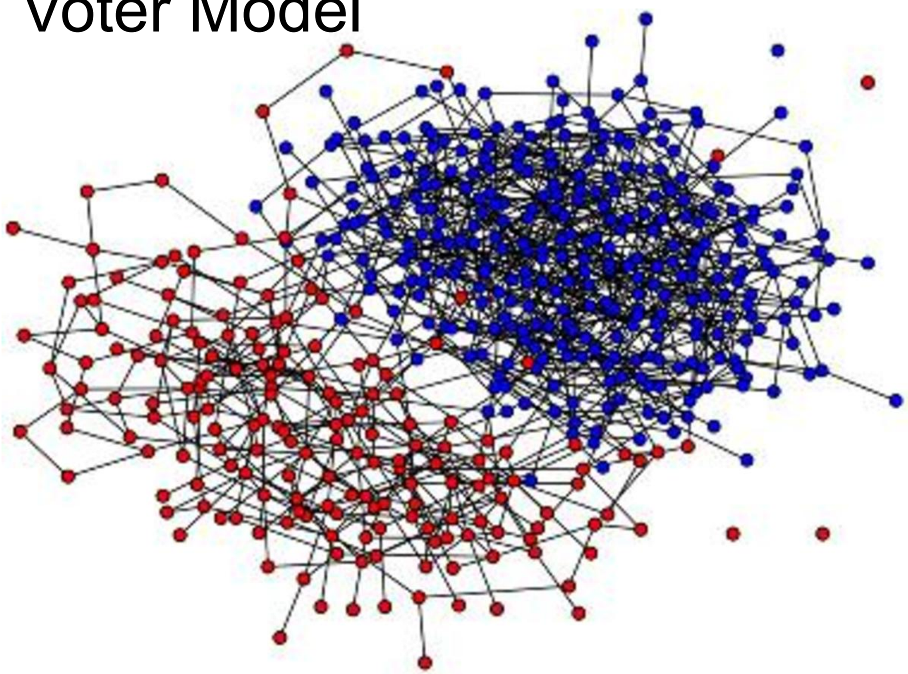
Funding thanks to the Watson Institute for Systems Excellence

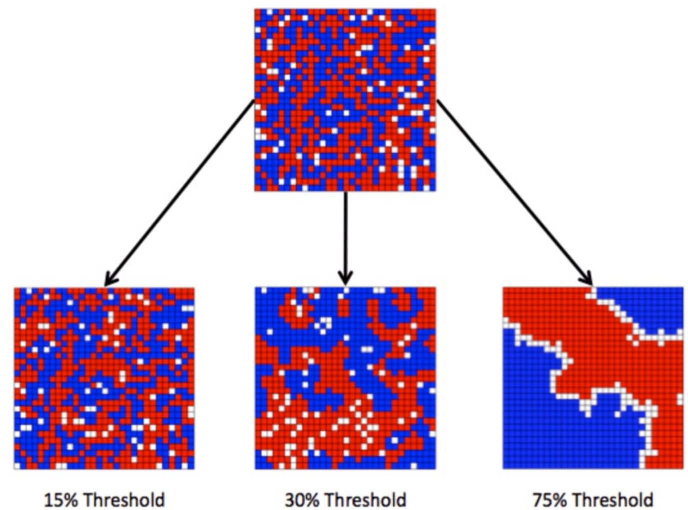

THE "MODERN" SOCIETY
Rock,Metal
Pop,Rap,Dupstap

Models of polarization (discrete)

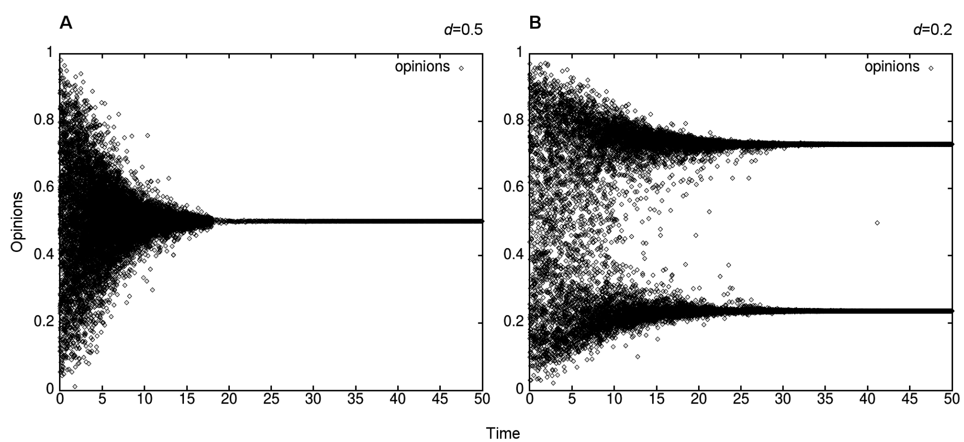
Voter Model



Schelling segregation



Deffaunt et al. 2000



More in *Social Physics*
Jusup et al. (arXiv)

Overview

- Derivation of continuous model
- Nonlocal interaction kernel
- Numerical simulation
- Exploration of parameter space

A continuous-field model

- Continuous population density: $\rho = \rho(x, t)$ (Sayama 2020)
  - $x =$ opinion space
  - $t =$ time

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$$

- $J =$ (Random movement) $+$ (Movement toward popular opinions)

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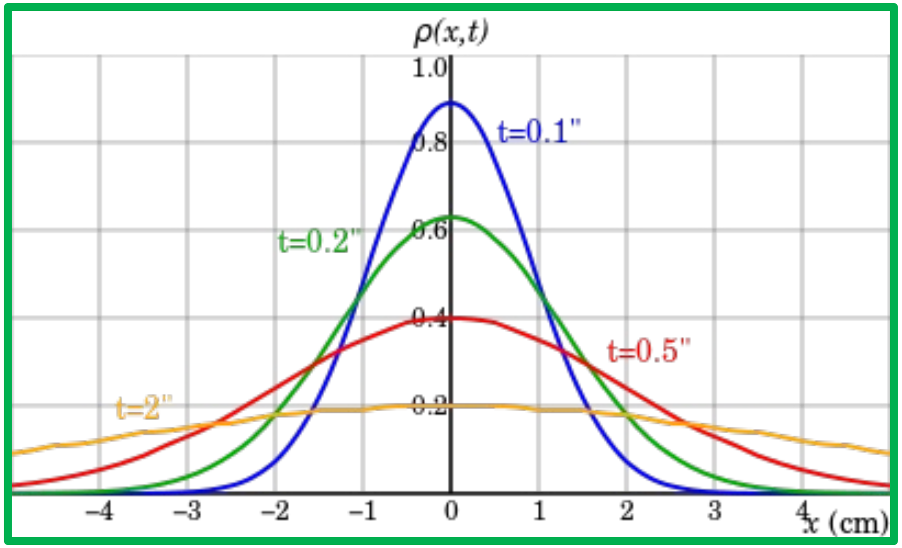

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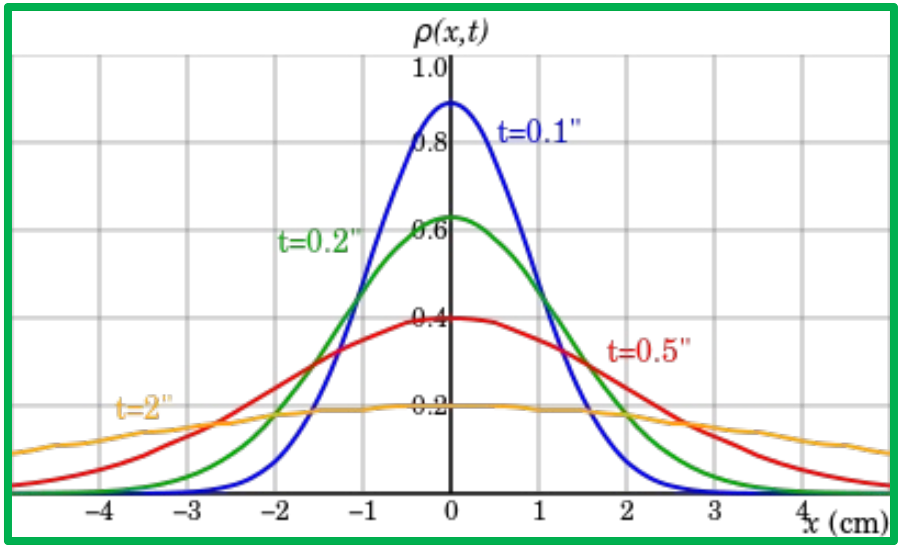

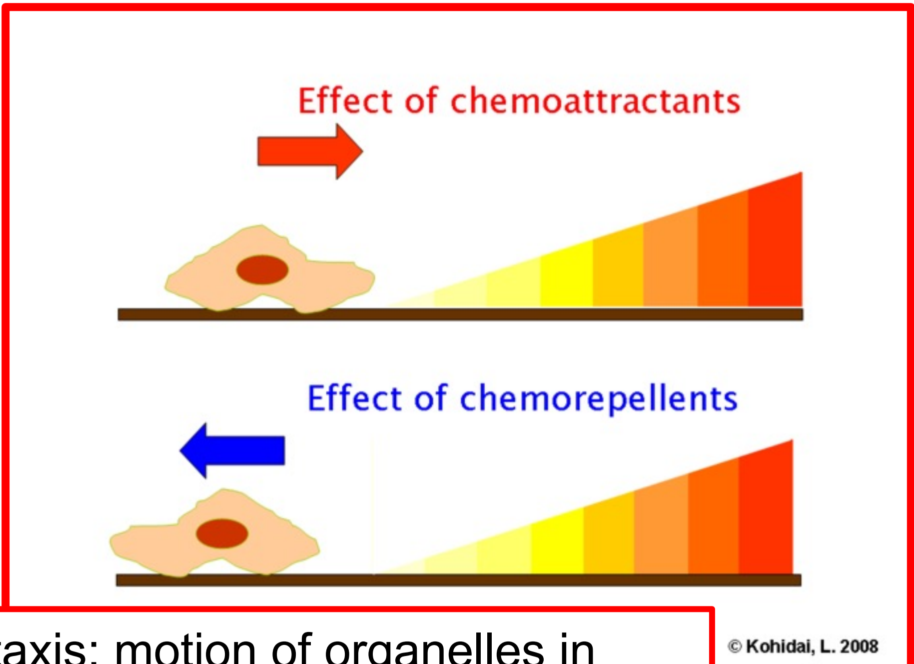

Chemotaxis: motion of organelles in response to chemical gradients (Keller and Segel 1970)

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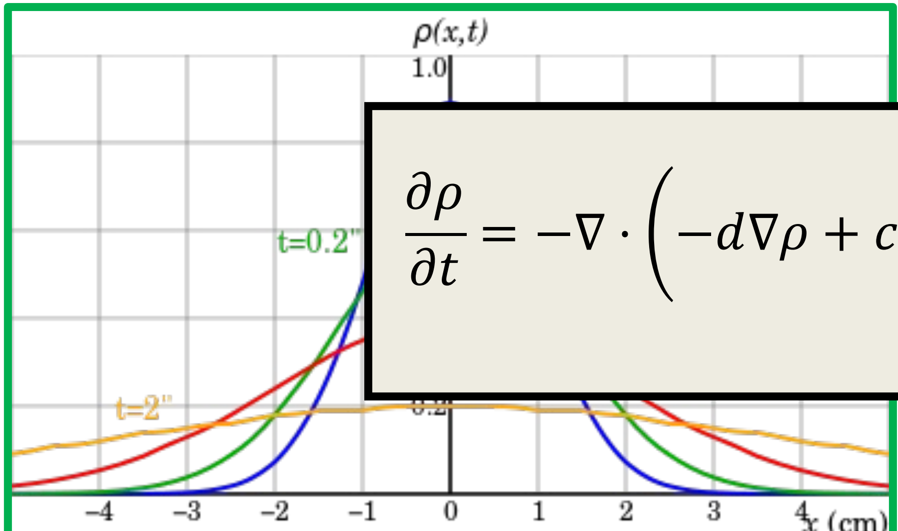
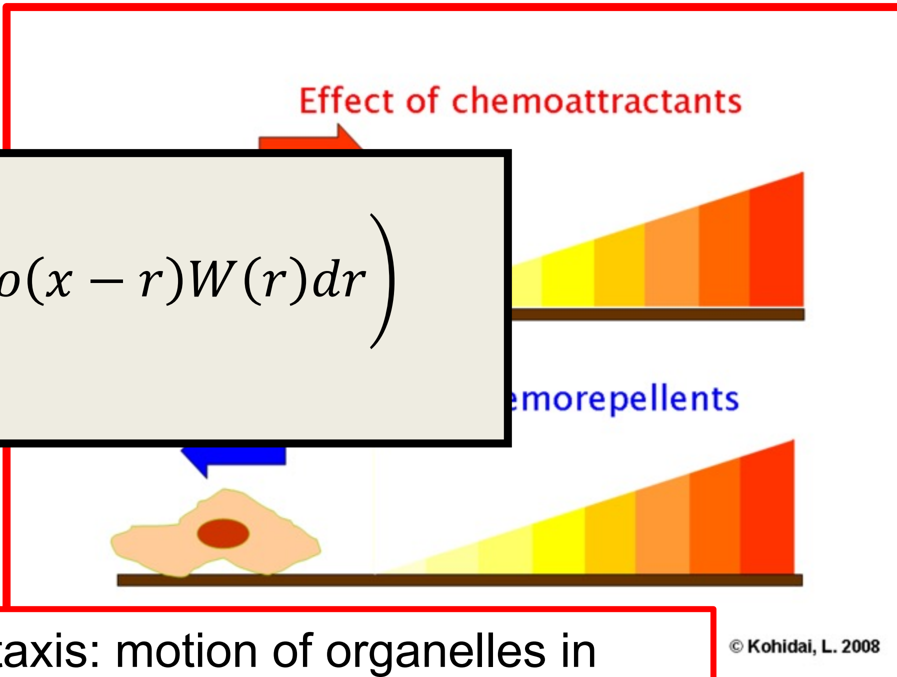


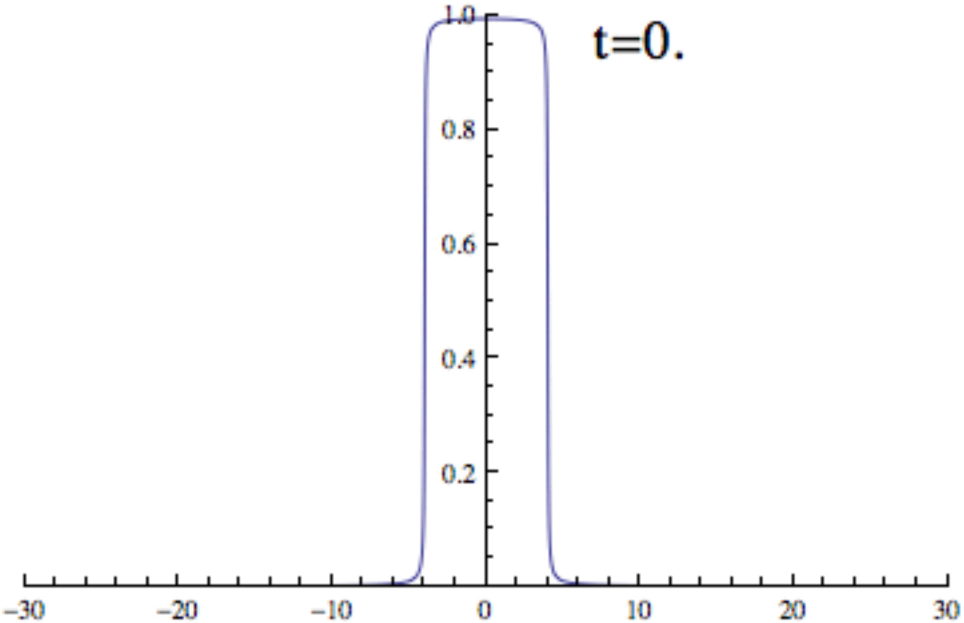
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(-d\nabla\rho + c\rho \int \rho(x-r)W(r)dr \right)$$

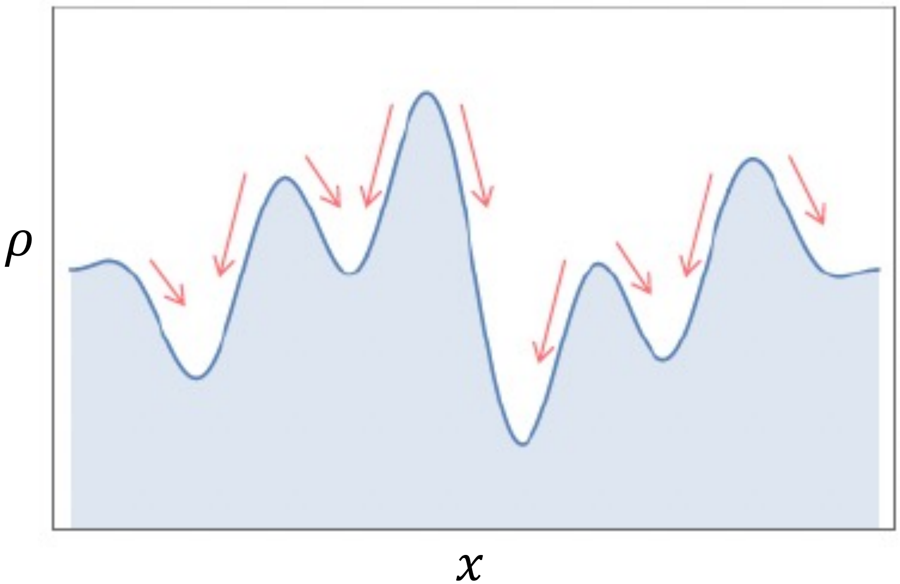


Chemotaxis: motion of organelles in response to chemical gradients (Keller and Segel 1970)

Interpreting the PDE (diffusion)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\underbrace{\boxed{-d\nabla\rho}}_{\text{Diffusion}} + c\rho \int \rho(x-r)W(r)dr \right)$$





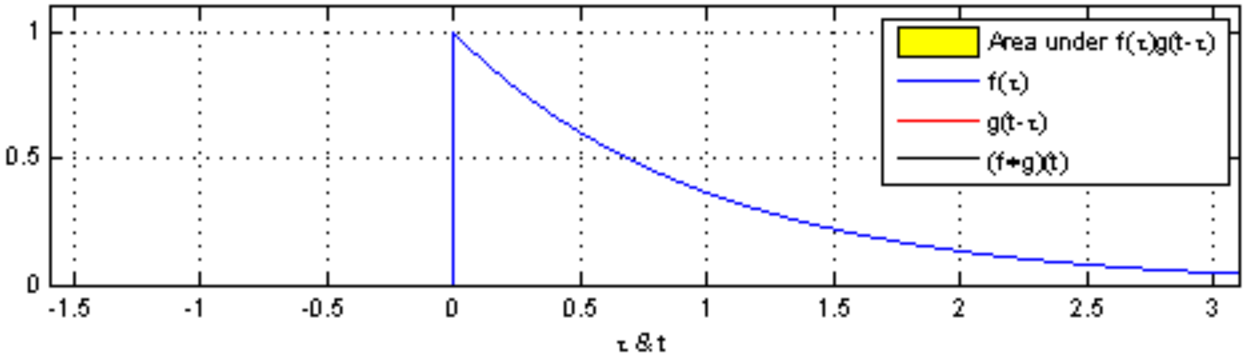
Credit: [kondensat]

Random motion

Interpreting the PDE

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(-d\nabla\rho + \boxed{c\rho \int \rho(x-r)W(r)dr} \right)$$

Nonlocal aggregation



Credit: [Brian Amberg]

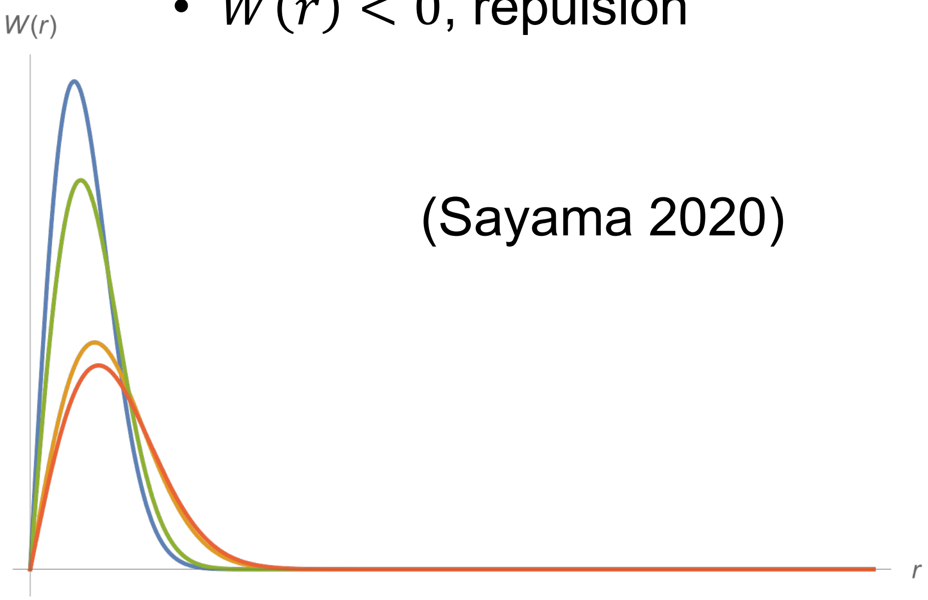
Long range sampling of the opinion space

Shapes of nonlocal interactions

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(-d\nabla\rho + c\rho \int \rho(x-r)\boxed{W(r)}dr \right)$$

Interaction kernel

- $W(r) \sim$ particle-based interaction potential
- For $r > 0$,
  - $W(r) > 0$, attraction
  - $W(r) < 0$, repulsion



(Sayama 2020)

Shapes of nonlocal interactions

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(-d\nabla\rho + c\rho \int \rho(x-r) \boxed{W(r)} dr \right)$$

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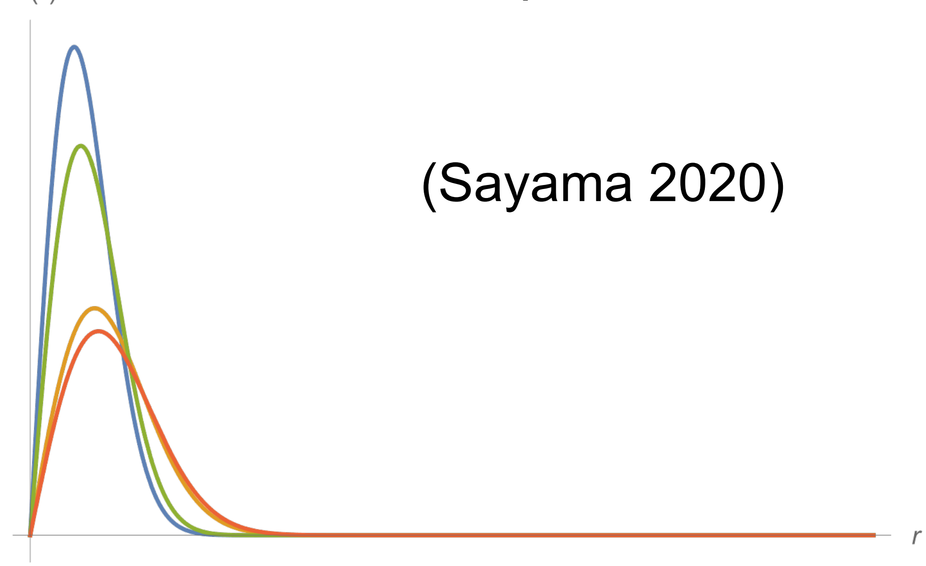
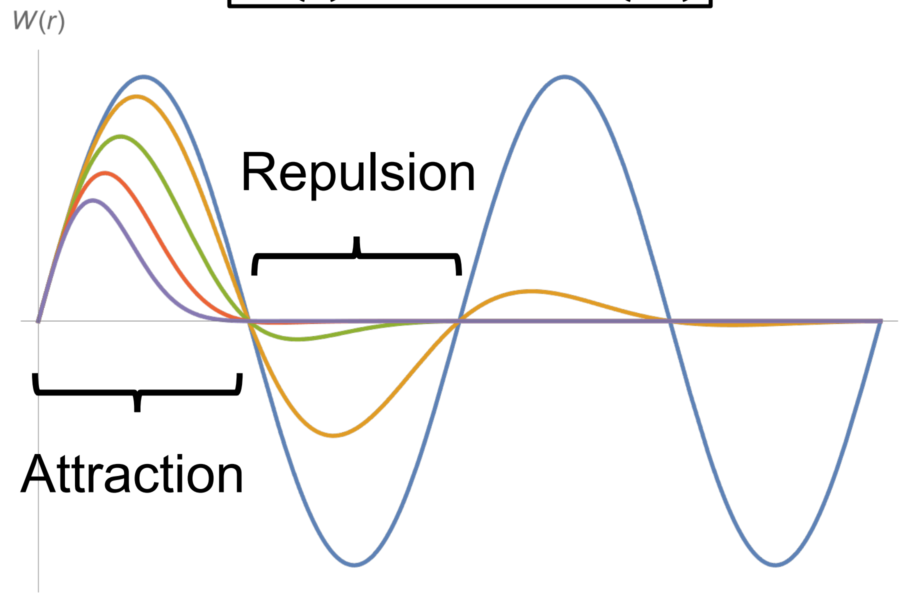
$\boxed{W(r) = e^{-\lambda^2 r^2} \sin(kr)}$ New

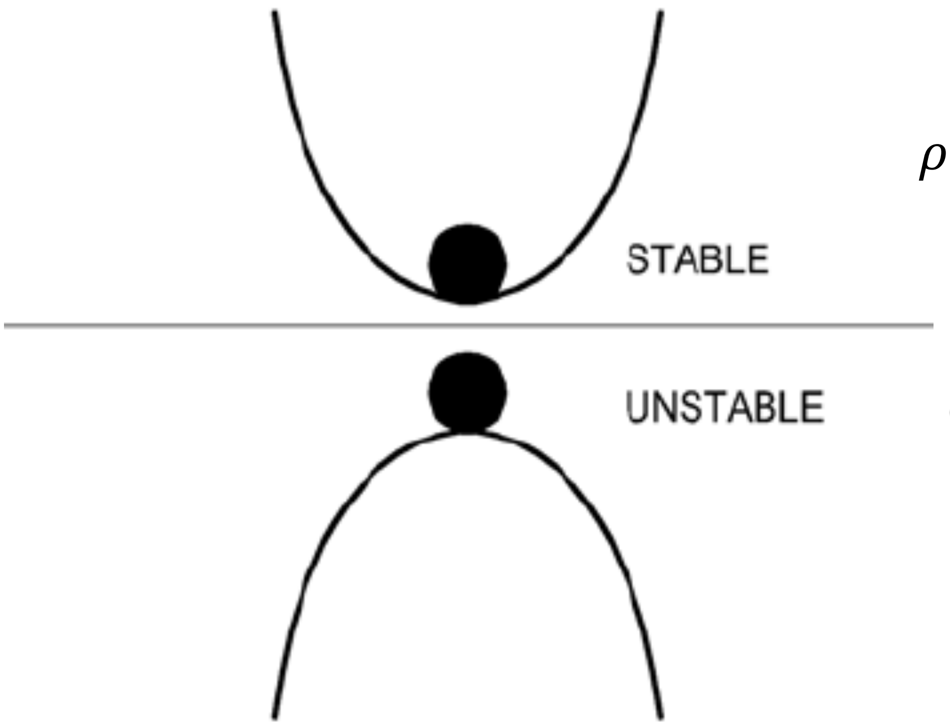


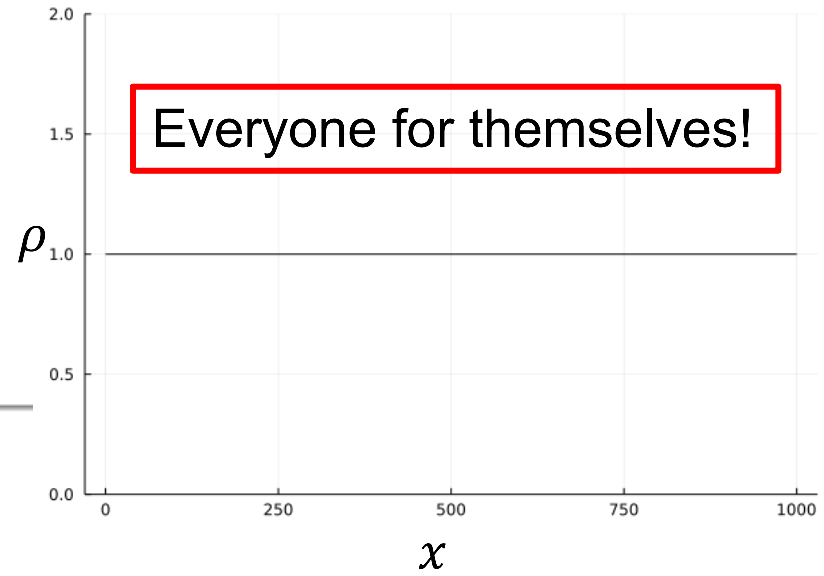
(Sayama 2020)



Onset of pattern formation

- Linear stability analysis: determines whether stationary states become unstable





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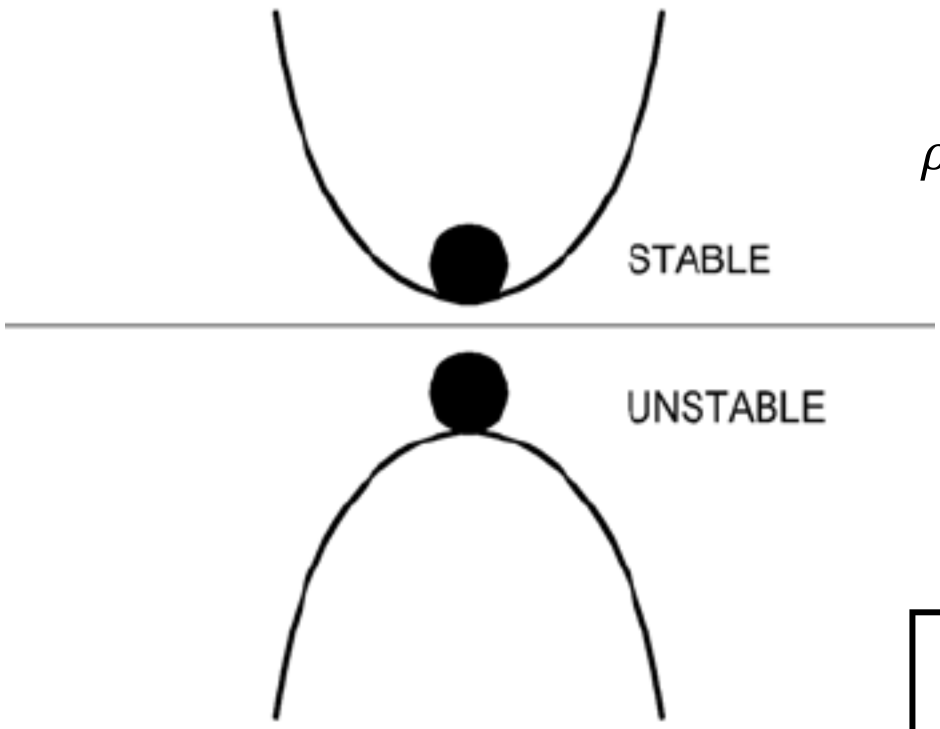
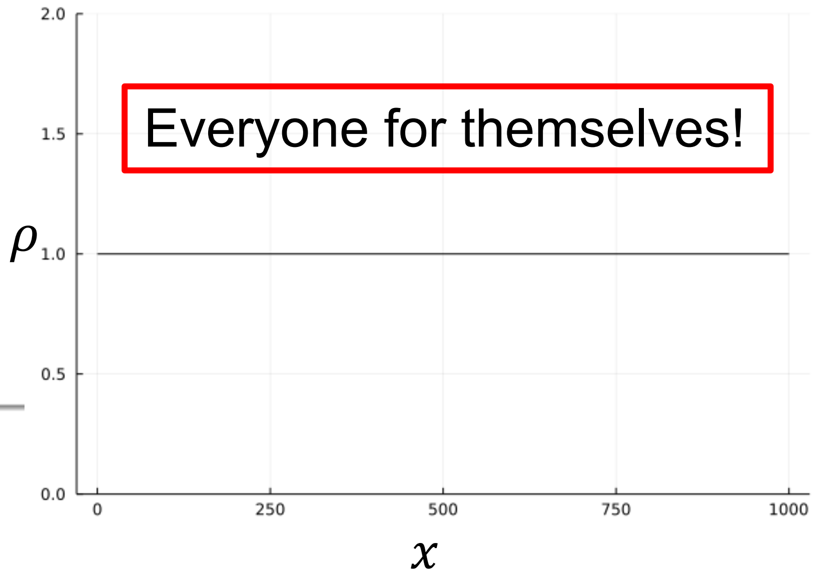


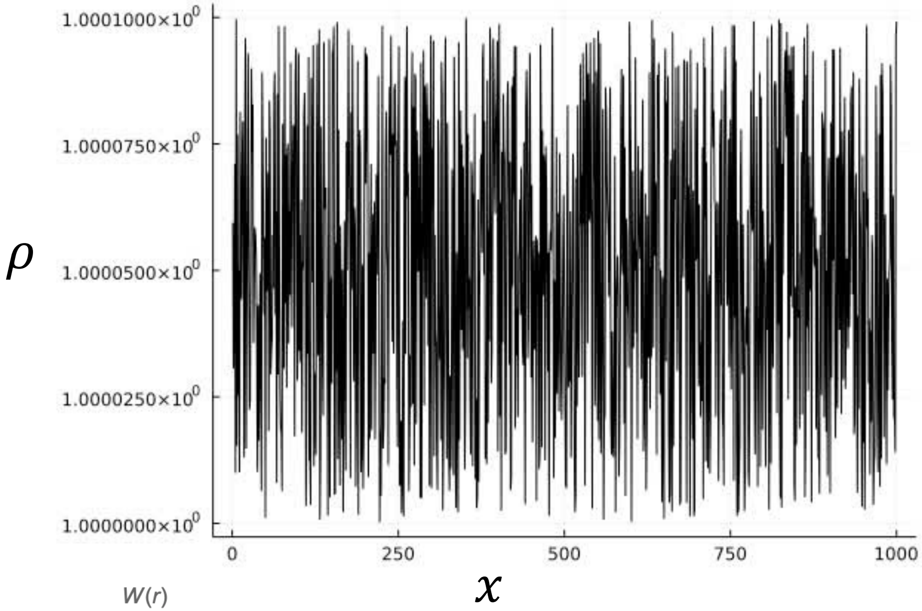


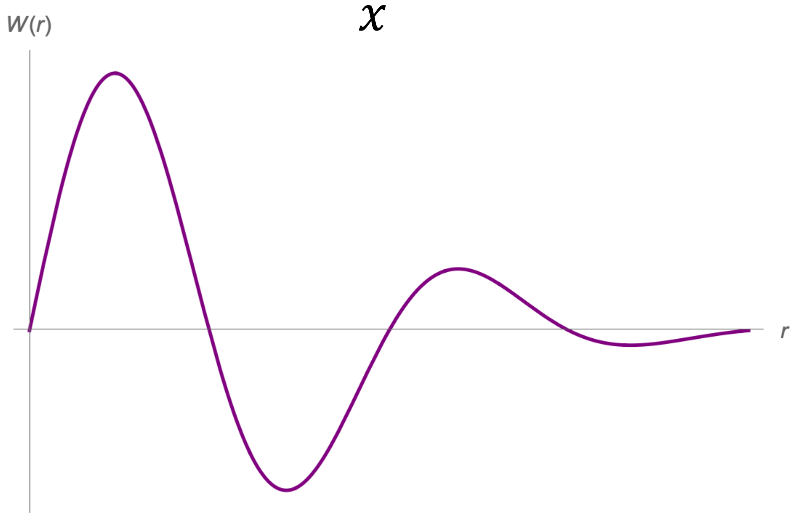
$$\rho(x,t) \to \rho_h + \Delta\rho(t)e^{ikx}$$

$$-dk^2 - ick\int W(y)e^{iky}dy > 0$$

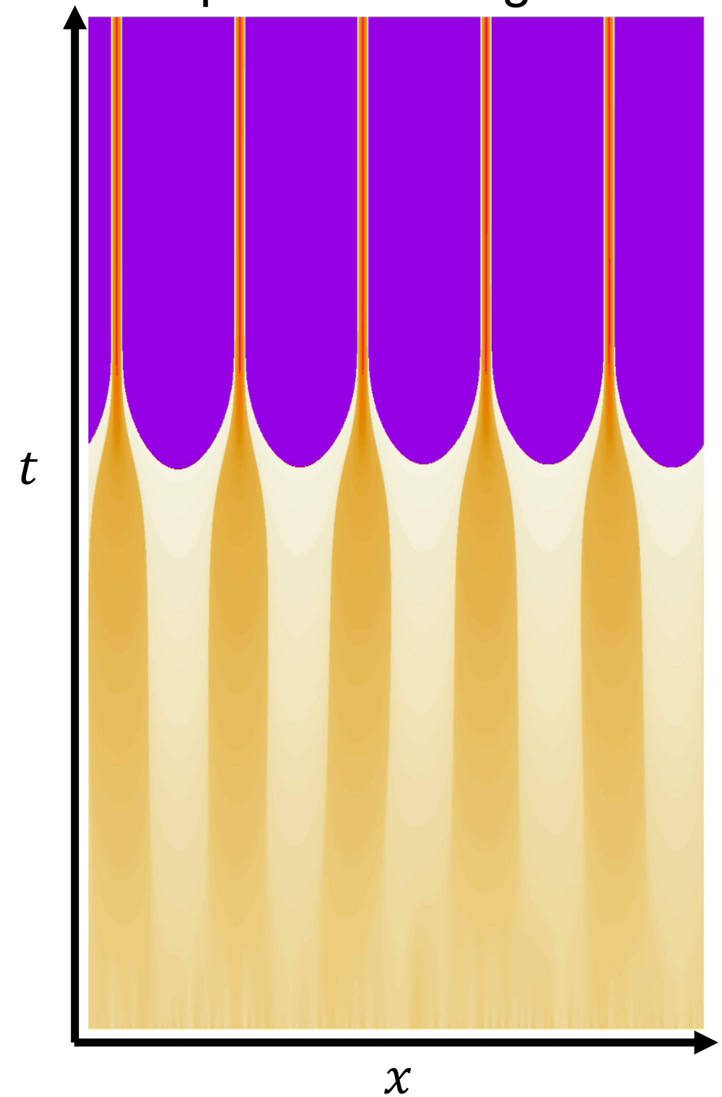
Simulation w/ periodic boundaries

- $\rho(x,t) = \rho(x+L,t)$



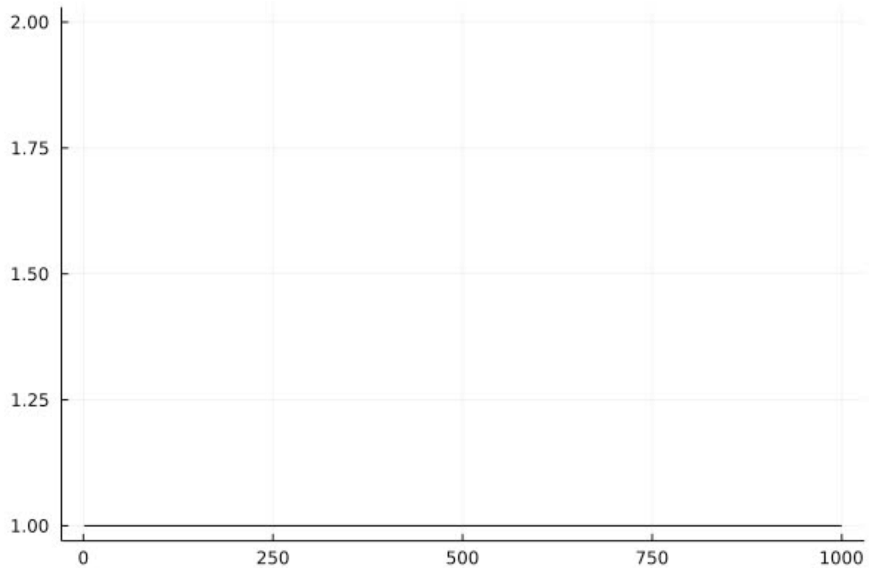


Spacetime diagram



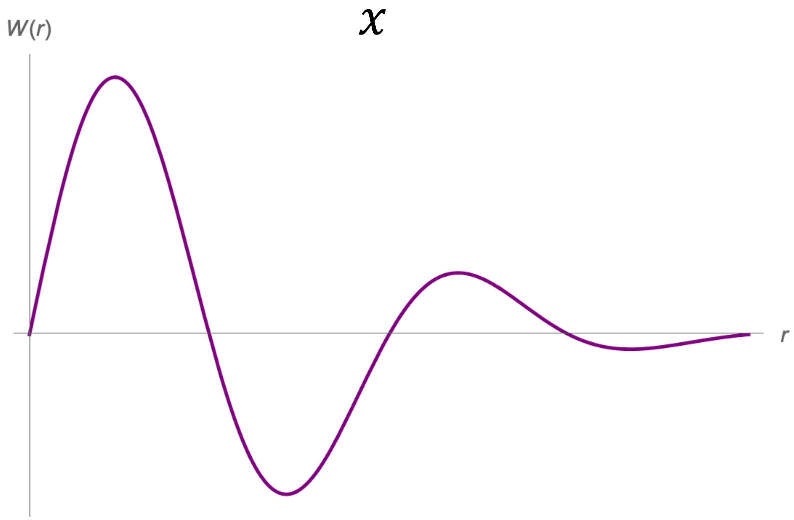
Simulation w/ hard boundaries

- $\rho(0,t) = \rho(L,t) = 0$



ρ

x



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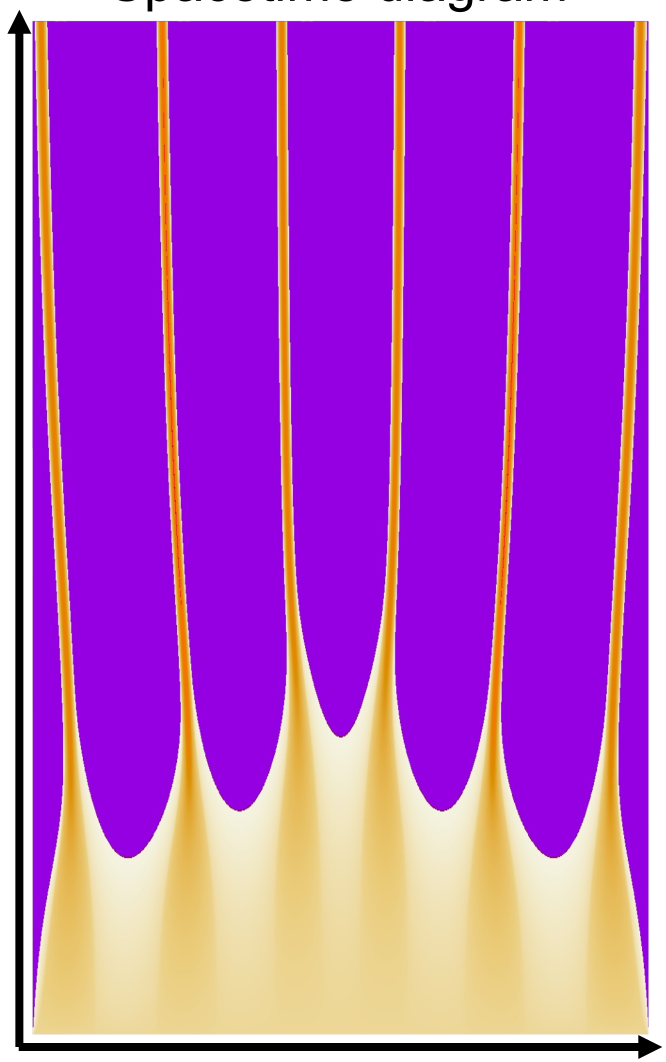
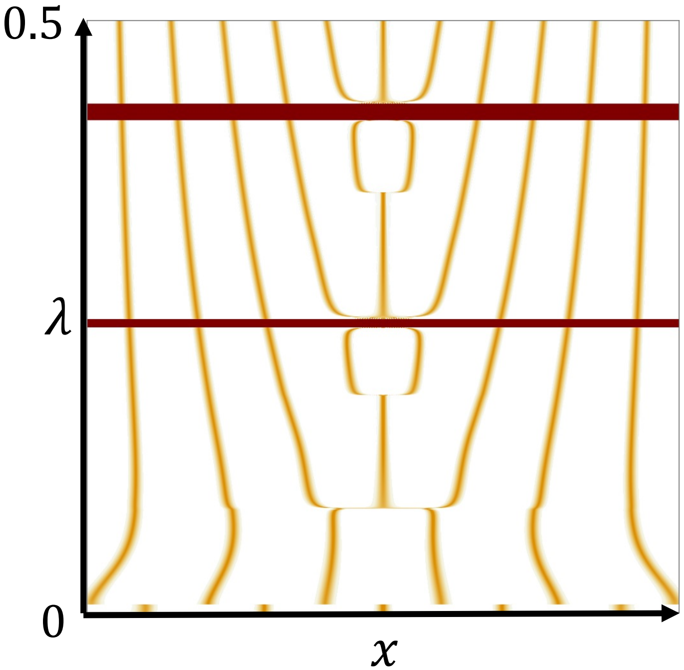


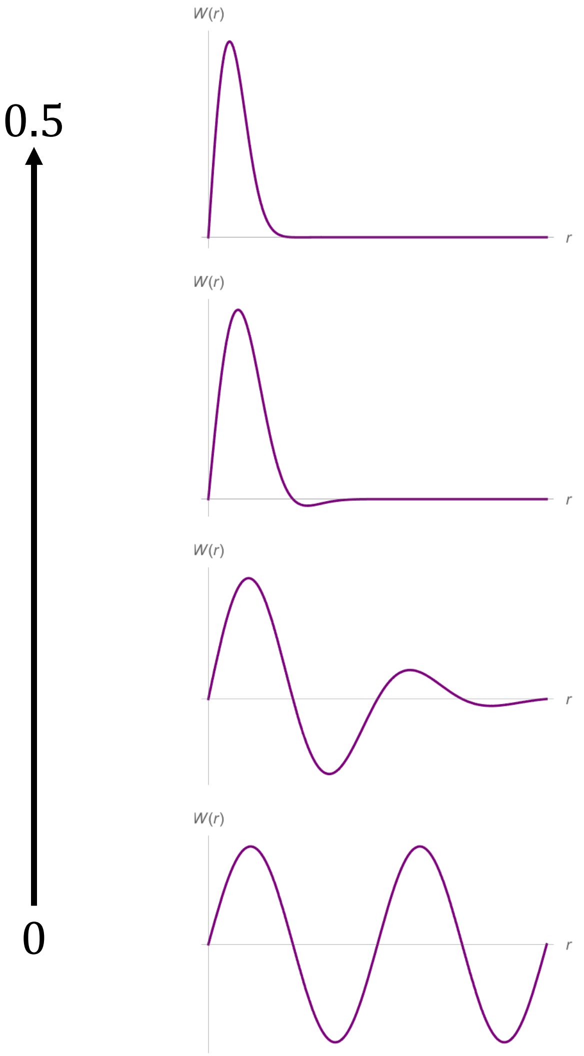
t

x

Parameter space

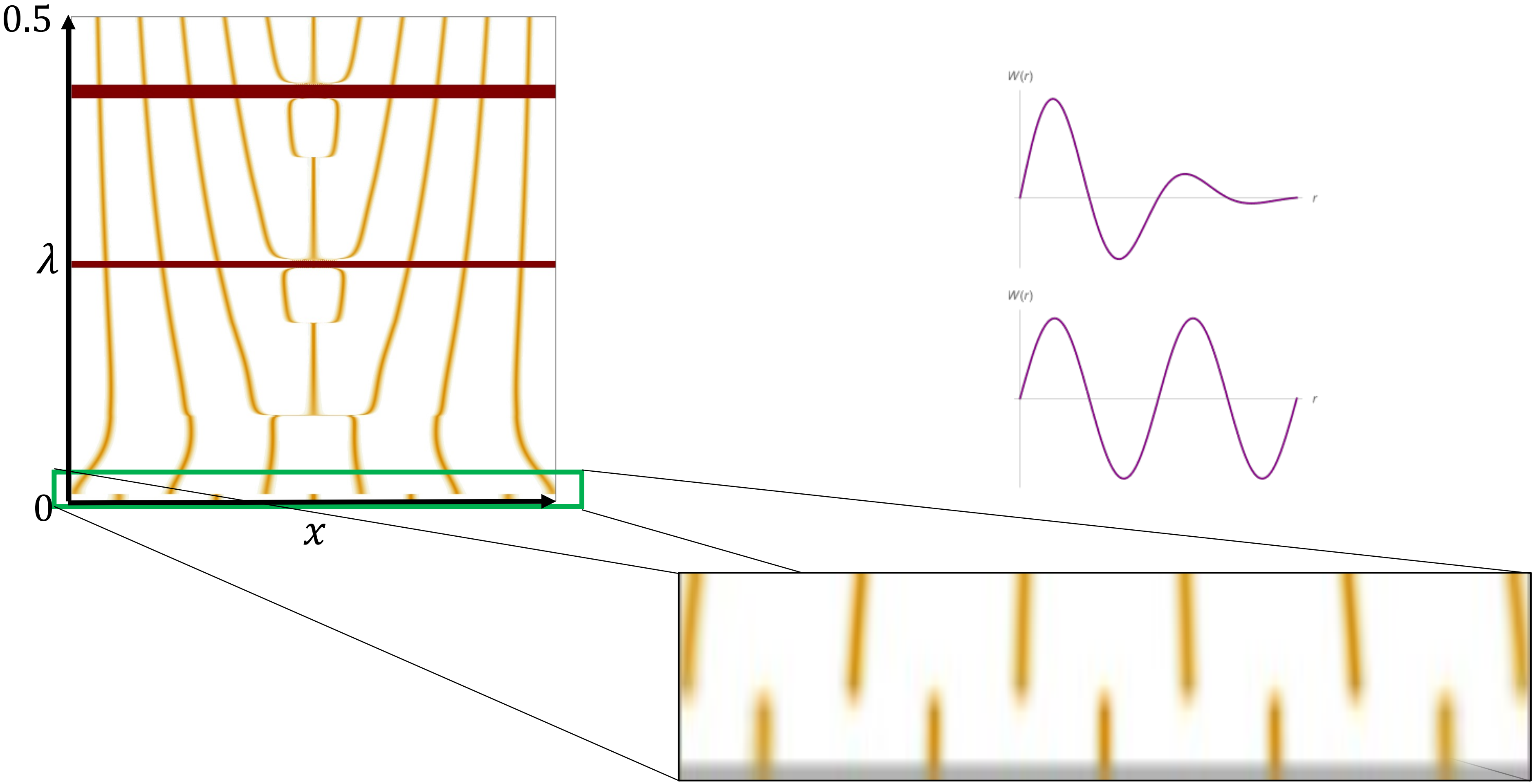


$$\boxed{W(r) = e^{-\lambda^2 r^2} \sin(kr)}$$



Parameter space

- Let’s zoom in,



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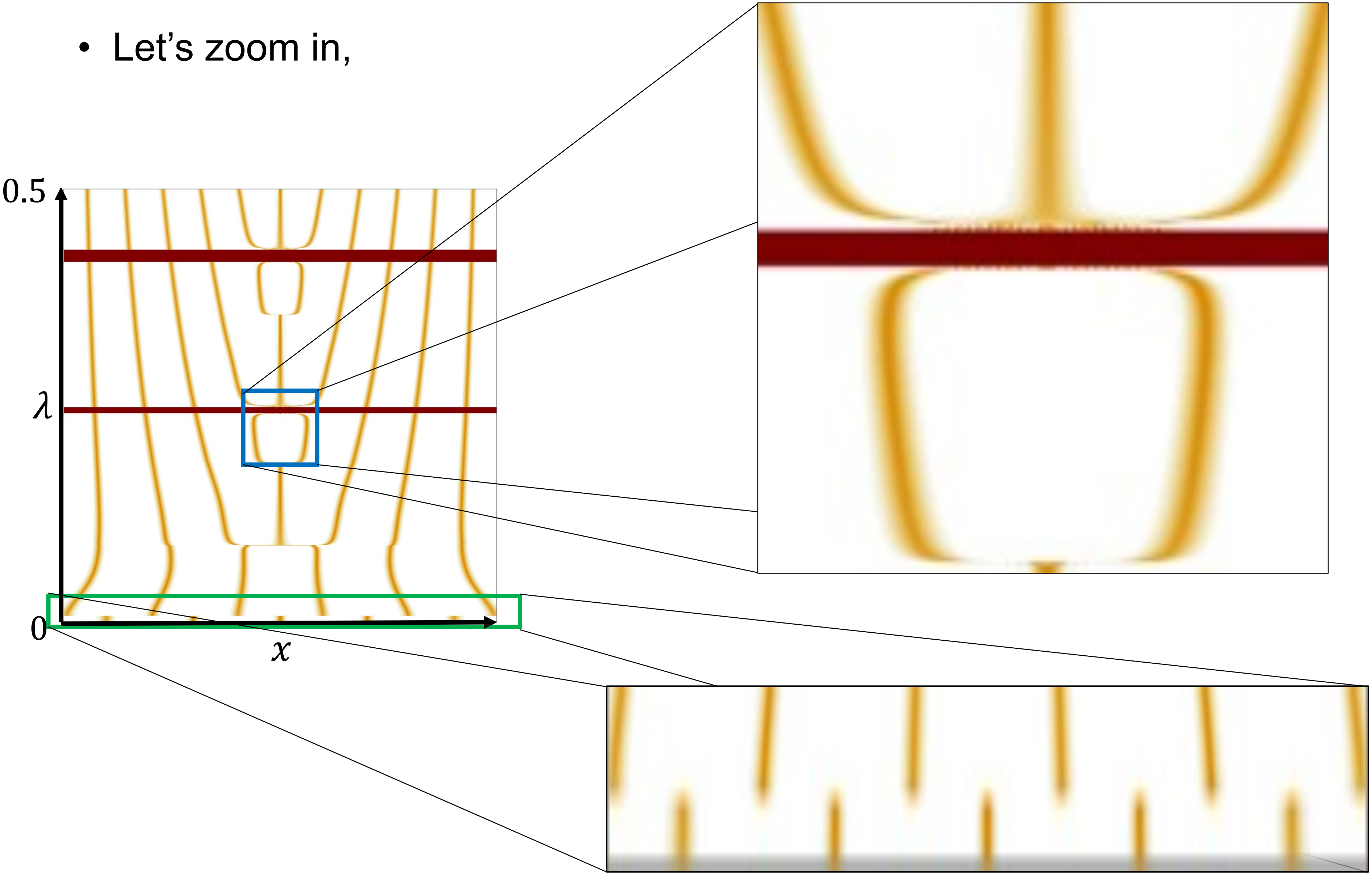

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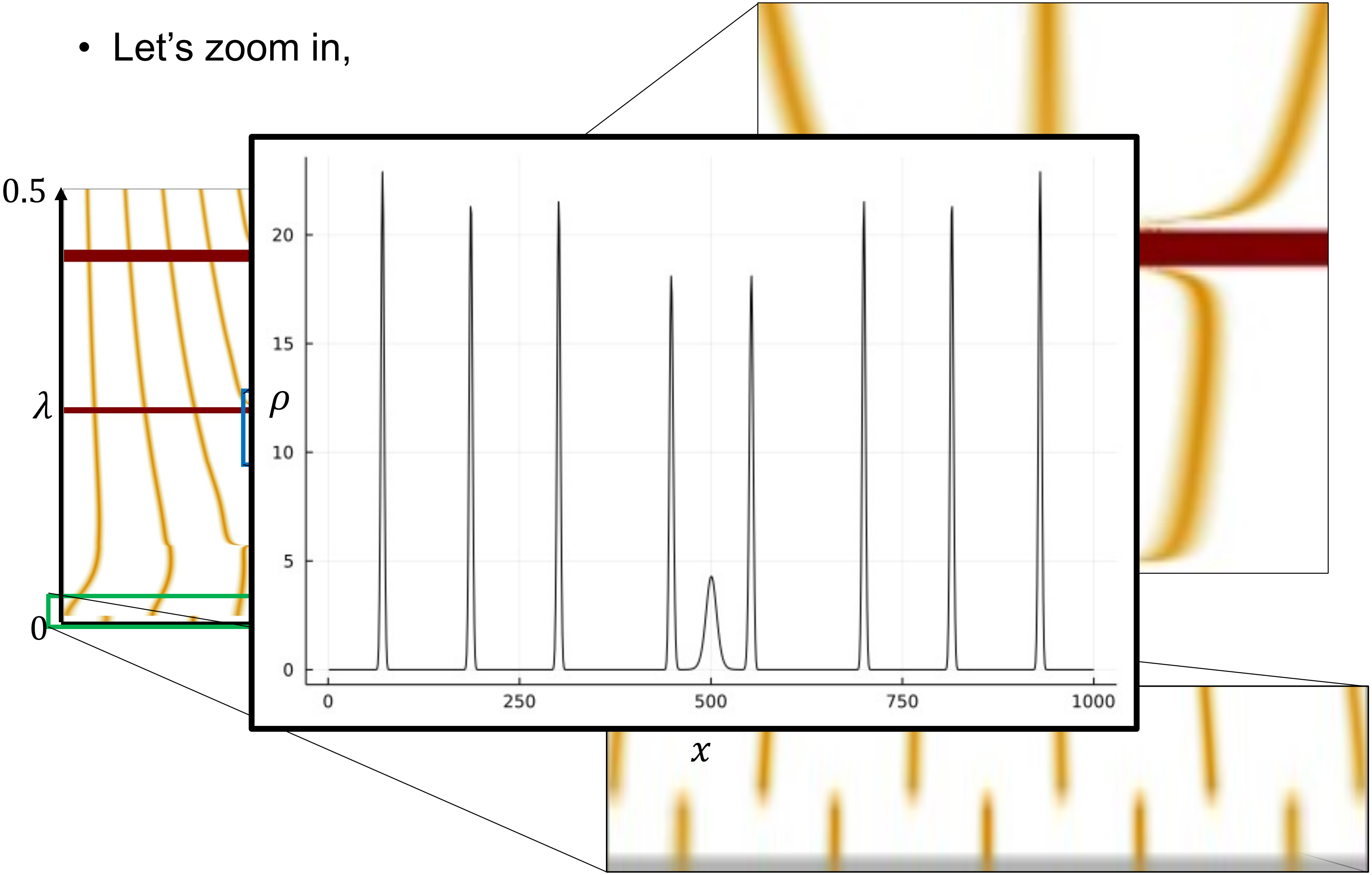
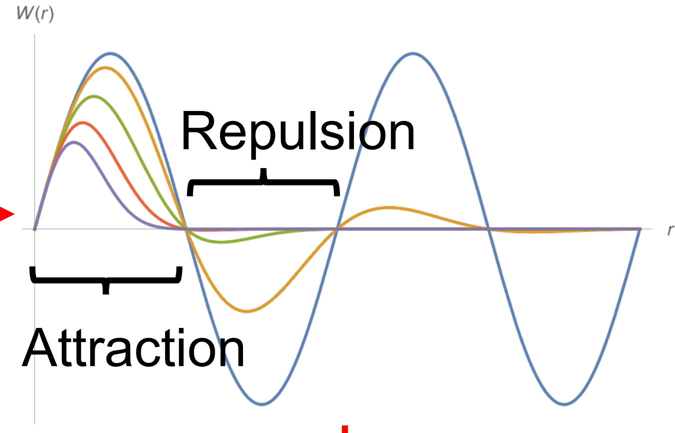
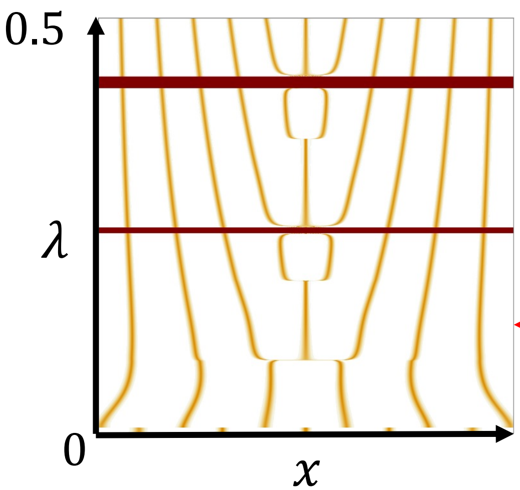
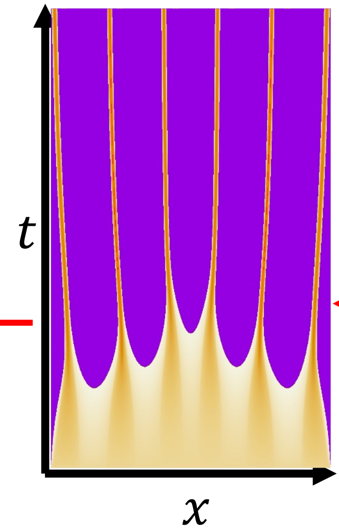


Discussion

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(-d\nabla\rho + c\rho \int \rho(x-r)W(r)dr \right)$$







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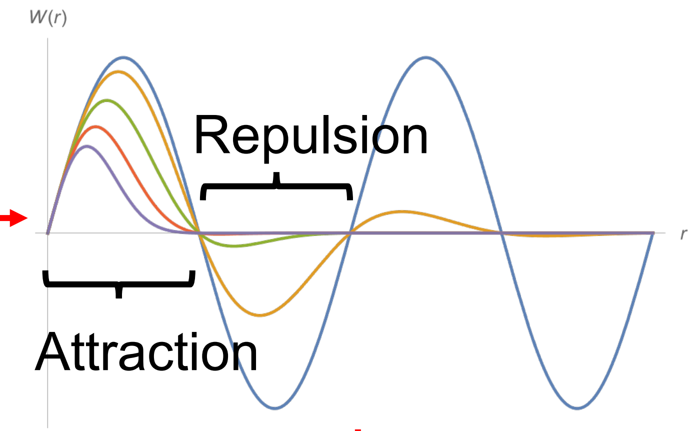

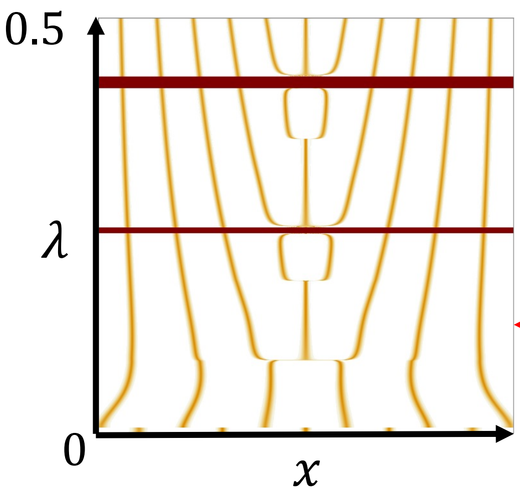

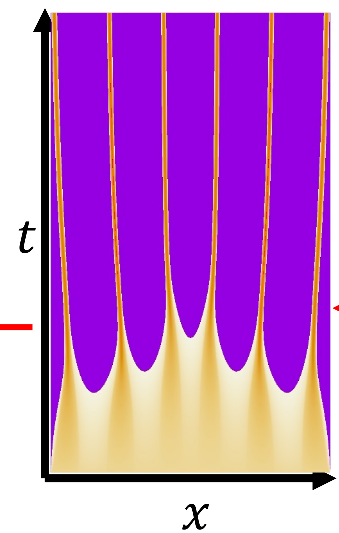

Future work

1. Are the opinion groups stable? (Multi-scale analysis of patterns)
2. Allow for varying total population (leading to spatiotemporal chaos)

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